# TEMPERATURE MEASUREMENT BY WALL-MOUNTED THERMOCOUPLES FOR POLYMER MELT FLOW-I. **THEORY**

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Abstract-A fully developed plane velocity profile locally independent of variations in the wall temperature is assumed to hold. Two cases are considered for the thermal boundary condition at the wall: (i) perfectly conducting, i.e. fixed temperature, and (ii) poorly conducting, i.e. temperature proportional to temperature gradient. The variation of wall temperature that occurs downstream of a change from condition (i) to condition (ii) is calculated as a function of distance downstream for large values of Péclet number. Its application to the measurement of the bulk temperature of flowing polymer streams is discussed.

# **NOMENCLATURE**

- $Ai(x)$ , Airy function (first kind);
- *Bi(x),*  Airy function (second kind);
- *Br,*  Brinkman number;
- $C_{P}$ , heat capacity of polymer;
- $f^{(n)}(z)$ , function defined by equation *(32);*

*k*  thickness of resin layer;

- *b.R,*  thermal conductivity of polymer, resin;
- *4*  length of resin layer;
- *P,*  Laplace transform variable;
- *PC,*  Peclet number;
- $\tau(P, \eta)$ , Laplace transform of  $\phi(\xi, \eta)$ ;
- *T*  temperature;
- $T^*(y)$ , developed temperature field;
- $T_W$ , wall temperature;
- *t,*  dummy variable;
- $U(a, b, z)$ , confluent hypergeometric function;
- $u$ , dimensionless velocity;
- $v_x$ , x-component of velocity;
- $x$ , axial coordinate;
- y, normal coordinate;<br>z similarity variable.
- similarity variable.

# Greek symbols

- $\alpha$ , thermal conductivity ratio;<br> $\dot{\Gamma}$ , shear rate;
- $\Gamma(x)$ , shear rate;<br> $\Gamma(x)$ , Gamma fui
- Gamma function;
- $\eta$ , dimensionless axial coordinate;<br> $\theta$ , dimensionless temperature;
- dimensionless temperature;
- A, dimensionless length of resin layer;
- $\lambda$ , scaled Péclet number;
- $\zeta$ , dimensionless axial coordinate;
- $\chi_0$ , temperature gradient at  $\xi = 0$ ;
- $\rho_P$ , polymer density;<br> $\zeta$ , stretched axial co
- 
- $\zeta$ , stretched axial coordinate,  $\zeta/Pe$ ;<br> $\psi$ , dimensionless excess temperature dimensionless excess temperature;
- $\psi$ <sub>0</sub>, dimensionless excess temperature, initial value.

# INTRODUCTION

ONE OF the more common and simple ways of measuring the temperature of a flowing polymer stream is by monitoring the voltage output of a thermocouple junction placed as close as possible to the boundary to the stream. In most cases this is a metal surface, and control of the polymer temperature is attempted by varying the heat input to or removal from the metal mass concerned.

However, polymer melts are usually processed in a very viscous state and significant temperature gradients are built up in the flowing material (of the order of 1 to  $10 K/mm$ ) because of high heat generation and relatively low thermal conductivity. This means that the temperature of the stagnant polymer material at the wall may well give a poor indication of the mean temperature of the polymer flowing through typical processing equipment.

We examine here the effect of varying the thermal conductivity of the wall material in the immediate environment of the temperature sensor and show that a pair of sensors can be used to provide information about heat fluxes and hence about temperature gradients in the polymer melt near the wall. This yields improved estimates of the bulk polymer temperature.

This analysis is given not so much because such a system is thought to be an ideal form of flux meter as because much equipment is already provided with wall-mounted thermocouples. Alternative devices for measuring the temperature of the polymer stream, remote from the wall, directly have proved to introduce large errors, difficult to calculate, of the order of the temperature difference they were intended to measure. These errors arise because sturdy metal sensors have to be used, which distort the flow and hence the pattern of heat generation, and also conduct heat away from the region close to the tip.

Preliminary measurements have already shown that differences of about 1°C can be measured by using a

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pair of sensors. one with a highly conducting and one with a poorly conducting face, placed in a plane metal wall bounding flow of a rubber melt.

# **MATHEMATICAL MODEI**

For simplicity we consider a plane unidirectional flow and plane symmetry. Figure I shows the geometry of the flow field and its boundaries. The velocity field



 $v_x(y)$  in the polymer melt is supposed to be steady and fully developed (independent of  $x$ ) and, to the approximation considered here, independent of small changes in the temperature field. Thus in the neighbourhood of  $y = 0$ , we assume that it can be well represented by a linear form (uniform shear).

$$
v_x(y) = \Gamma y \tag{1}
$$

which applies for all  $x$ . This is consistent with a constant viscosity  $\eta_P$  for the polymer melt (independent of temperature).

The temperature field is also supposed to be steady and fully developed far from the line  $x = 0$ ; for  $x < 0$ (or more strictly, for  $x \to -\infty$ ), we take

$$
T = T^*(y)
$$

where  $T^*$  is determined by the local balance between conduction and generation of heat, and by  $T^*(0) = T_0$ , the (constant) temperature of the metal wall. For  $x > 0$ , the wall boundary condition on the temperature changes because a layer of poorly conducting material (resin say) of thickness  $h$  is interposed between the flowing polymer and the metal. We have shown the resin layer to apply only for  $0 \le x \le l$ , as would be the case in practice, but for calculation purposes we shall suppose that  $l \gg h$ . The problem is to calculate the wall temperature distribution  $T_W = T(x, y = 0)$  given that the thermal conductivity of polymer and resin is low.

In general, the steady temperature field would be given by

$$
\rho_P C_P v_x(y) \frac{\partial T}{\partial x} = k_P \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \eta_P \left( \frac{\partial v_x}{\partial y} \right)^2 \quad (2)
$$

for the flowing melt region  $y > 0$ , and

$$
\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = 0\tag{3}
$$

for the rigid resin region  $-h \leq y \leq 0$  and  $0 \leq x \leq l$ .

The boundary condition at the (perfectly conducting) metal boundary is

$$
T = T_0 \quad \text{for} \quad \begin{cases} y = 0, & x < 0 \quad \text{and} \quad x > l \\ y = -h, & 0 \le x \le l. \end{cases} \tag{4}
$$

T and  $k(\partial T/\partial y)$  must be continuous at the polymer melt/resin interface  $y = 0$ ,  $0 \le x \le l$  i.e.

$$
k_P \left( \frac{\partial T}{\partial y} \right)_{y=0+} = k_R \left( \frac{\partial T}{\partial y} \right)_{y=0-} \quad \text{for} \quad 0 < x < l. \tag{5}
$$

Far from the region in the melt disturbed thermally by the resin region, the temperature field will be the unperturbed temperature field. i.e.

$$
T = T^*(y) \quad \text{for} \quad x \to \pm \infty \quad \text{and} \quad y \to +\infty \quad (6)
$$

By substitution into equation (2),  $T^*(y)$  is seen to obey the equation

$$
k_P \frac{\partial^2 T^*}{\partial y^2} + \eta_P \left(\frac{\partial v_x}{\partial y}\right)^2 = 0 \quad \text{with} \quad T^*(0) = T_0. \tag{7}
$$

For  $\partial T^*/\partial y$  and  $T^*$  to be bounded, certain restrictions have to be imposed on  $v_x(y)$  but we need not be more precise at this stage because we shall only use (7) in the neighbourhood of  $y = 0$ , when (1) applies. Here  $\rho_P$ ,  $C_P$ ,  $\eta_P$  and  $k_P$  are the density, specific heat, viscosity and thermal conductivity respectively of the polymer melt, and  $k_R$  is the thermal conductivity of the resin,

# **DIMENSIONLESS REPRESENTATION; HIGH PÉCLET NUMBER APPROXIMATION**

We use new variables

$$
u = v_x/\dot{\Gamma}h, \quad \xi = x/h, \quad \eta = y/h, \quad \theta = (T - T_0)b \quad (8)
$$

where  $b^{-1}$  is some so far undefined scale temperature. Equations (2) and (3) then become

$$
Pe u \frac{\partial \theta}{\partial \xi} = \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}\right) + Br\left(\frac{\partial u}{\partial \eta}\right)^2, \quad \eta > 0, \qquad (9)
$$

$$
0 = \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}\right), \quad -1 < \eta < 0, \quad 0 \le \xi \le \Lambda \tag{10}
$$

where

$$
Pe = \frac{\Gamma h^2 \rho_P C_P}{k_P}, \quad Br = \frac{\eta_P \Gamma^2 h^2 b}{k_P} \quad \text{and} \quad \Lambda = l/h. \tag{11}
$$

The boundary conditions become

$$
\theta = 0 \quad \text{for} \quad \begin{cases} \eta = 0 & \xi < 0 \quad \text{and} \quad \xi > \Lambda \\ \eta = -1 & 0 \le \xi \le \Lambda \end{cases} \tag{12}
$$

$$
[\theta]_{\eta=0} = 0, \quad \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0+} = \alpha \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0-} \tag{13}
$$

where

$$
\alpha = k_R / k_P \tag{14}
$$

and

$$
\theta \to \theta^*(\eta) \quad \text{for} \quad \eta \to \infty, \quad \xi \to \pm \infty. \tag{15}
$$

We have already noted that  $\Lambda \gg 1$ . We now assert that in most cases of interest  $Pe \gg 1$  also [1]. This implies that a thin thermal boundary layer develops upstream of the line  $x = 0$  over the surface  $y = 0$ .

Br becomes a measure of the importance of We now use the similarity variable generation, insofar as coupling between velocity and temperature fields is concerned. We assume

$$
Br \leq 1\tag{16}
$$

and  $\alpha$  we take to be of order unity.

We now take a stretched variable

$$
\zeta = \xi / Pe \tag{17}
$$

and neglect terms of order  $Pe^{-1}$  etc. We then obtain the much simpler equations

$$
\eta \frac{\partial \theta}{\partial \zeta} = \frac{\partial^2 \theta}{\partial \eta^2} + Br; \ \eta > 0 \tag{18}
$$

$$
0 = \frac{\partial^2 \theta}{\partial \eta^2}; \ -1 < \eta < 0, \ \ 0 \leqslant \zeta \leqslant \lambda \tag{19}
$$

where  $\lambda = l/h$  Pe, and the limiting form (1) has been used for u. The boundary conditions are unchanged. If we now write To satisfy the boundary condition (23 iii), we shall

$$
\theta = \theta^*(\eta) + \phi(\zeta, \eta) \tag{20} \qquad \text{require}
$$

then  $\phi$  obeys the equations

$$
\eta \phi_{\text{polymer}} = \phi_{\text{polymer}\eta\eta}; \quad \eta \geqslant 0 \tag{21}
$$

$$
0 = \phi_{\text{resin}\eta\eta}; \quad -1 < \eta \leq 0, \quad 0 \leq \zeta \leq \lambda \qquad (22)
$$

with boundary conditions

$$
\phi_{\text{resin}} = 0 \quad \eta = -1 \quad 0 \le \zeta \le \lambda \quad \text{(i)}
$$
\n
$$
\phi_{\text{polymer}} = 0 \quad \left\{ \eta = 0 \quad \zeta \le 0, \zeta \ge \lambda \quad \text{(ii)}
$$
\n
$$
\phi_{\text{polymer}} = 0 \quad \eta \to \infty \quad \text{all } \zeta \quad \text{(iii)}
$$
\n(23)

 $\zeta \rightarrow +\infty$ 

$$
\phi_{\text{polymer}} = \phi_{\text{resin}}, \quad \frac{\partial \phi_{\text{polymer}}}{\partial \eta} = \alpha \frac{\partial \phi_{\text{resin}}}{\partial \eta} - \chi_0
$$
\n
$$
\text{at } n = 0 \quad 0 < \zeta < \frac{1}{2} \tag{24}
$$

where

$$
\chi_0 = \left(\frac{\partial \theta^*}{\partial \eta}\right)_{\eta=0}.\tag{25}
$$

 $(iv)$ 

Here the disturbance field  $\phi$  has been separately labelled for the polymer melt and resin regions,

In practice, the parameter  $\lambda$  plays no part in the solution obtained, but is included in order to reflect the actual physical circumstances usually relevant.

$$
\theta^*(\eta) = \chi_0 \eta - \frac{1}{2} Br \eta^2 \tag{26}
$$

for  $\eta$  small enough, and using the definition (25).

# **SIMILARITY SOLUTIONS AND SERIES EXPANSION IN**  $\zeta$

If we write

$$
\phi_{\text{resin}}(\zeta,0) = \phi_0(\zeta),\tag{27}
$$

then (22), using (23i), has solution

$$
\phi_{\text{resin}} = \phi_0(1+\eta); \quad -1 \leq \eta \leq 0 \tag{28}
$$

and

$$
\alpha \Big(\frac{\partial \phi_{\text{resin}}}{\partial \eta}\Big)_{\eta=0-} = \alpha \phi_0.
$$

$$
z = \eta(9\zeta)^{-1/3} \tag{29}
$$

as is usual in such thermal boundary layer problems in terms of which (21) becomes

$$
\phi_{\text{polymer }zz} + 3z^2 \phi_{\text{polymer }z} = 9z \zeta \phi_{\text{polymer }z}.
$$
 (30)

The boundary condition (24) then becomes

$$
\phi_{\text{polymer }z}(0) = \zeta^{1/3} 9^{1/3} (\alpha \phi_0 - \chi_0). \tag{31}
$$

We look for a solution in the form

$$
\phi_{\text{polymer}} = \sum_{n=1}^{\infty} F_n f^{(n)}(z) \zeta^{n/3} \tag{32}
$$

where each of the terms  $f^{(n)}(z)$  are similarity solutions of (30) obeying the relation

$$
f_{zz}^{(n)} + 3z^2 f_z^{(n)} = 3n z f^{(n)}.
$$
 (33)

$$
f^{(n)}(z) \to 0 \quad \text{as} \quad z \to \infty \,. \tag{34}
$$

Without loss of generality we take

$$
f^{(n)}(0) = 1.
$$
 (35)

It can readily be shown that solutions to (33) subject to  $(34)$  and  $(35)$  are given by

$$
f^{(n)}(z) = e^{-z^3} \frac{\Gamma(1 + \frac{1}{3}n)}{\Gamma(\frac{1}{3})} U(\frac{2}{3} + \frac{1}{3}n, \frac{2}{3}, z^3)
$$
 (36)

where  $U(a, b, z)$  is the confluent hypergeometric function as defined by Slater [2]. We are grateful to Dr. T. J. Pedley for pointing this out to us.

It may readily be shown also that

$$
f_{z}^{(n)}(0) = -\frac{\Gamma(1+\frac{1}{3}n)\Gamma(\frac{2}{3})}{\Gamma(\frac{2}{3}+\frac{1}{3}n)\Gamma(\frac{4}{3})}.
$$
 (37)

We now substitute (32) into (31) and use (37) to get

$$
\sum_{n=1}^{a} F_n f_z^{(n)}(0) \zeta^{n/3} = 9^{1/3} \zeta^{1/3} \left( \alpha \sum_{n=1}^{\infty} F_n \zeta^{n/3} - \chi_0 \right) (38)
$$

From (18) we see that and, by equating successively the terms in  $\zeta^{1/3}$ ,  $\zeta^{2/3}$ ,...

$$
F_1 = \frac{9^{1/3}}{\Gamma(\frac{2}{3})} \chi_0 \tag{39}
$$

$$
F_2 = \frac{9^{1/3} \alpha F_1}{f_z^{(2)}(0)} = -\frac{3\alpha \chi_0 \Gamma^2(\frac{1}{3})}{2\Gamma(\frac{2}{3})^{3} 9^{1/3}}.
$$
 (40)

etc.

Hence we obtain

$$
\phi_0 = \frac{(9\zeta)^{1/3}\chi_0}{\Gamma(\frac{2}{3})} \left( 1 - \frac{(9\zeta\alpha^3)^{1/3}\{\Gamma(\frac{1}{3})\}^2}{6\{\Gamma(\frac{2}{3})\}^2} + (-1)^n(\frac{1}{3}\zeta\alpha^3)^{n/3} \frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{1}{3}n + \frac{4}{3})} \left\{ \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}\right\}^n + \dots \right) \quad (41)
$$

It is clear that this expression only converges for **REFERENCES REFERENCES** 

$$
\zeta \alpha^3 < 1 \tag{42}
$$

and is only useful when  $9\zeta \alpha^3 = 9\alpha^3 \zeta / Pe$  is significantly smaller than unity. An alternative derivation of (41) is 2. L. J. Slater, Confluent hypergeometric functions, in

From (41) we can write the wall temperature as

$$
T_W(x) = T_0 + h \left(\frac{\partial T^*}{\partial y}\right)_0 \left(\frac{9x}{hPe}\right)^{1/3} \frac{1}{\Gamma(\frac{2}{3})}
$$

$$
\left(1 - \frac{k_R}{k_P} \left(\frac{9x}{hPe}\right)^{1/3} \frac{\{\Gamma(\frac{1}{3})\}^2}{6\{\Gamma(\frac{2}{3})\}^2} + h.o.t.\right). \quad (43)
$$

A particularly simple result follows if the resin is  $\frac{1}{16}$  very large. regarded as a heat insulator,  $\alpha \rightarrow 0$ . We can then compare the contribution to  $T_w(x)$  given by the term  $T(P, \eta) = \int_0^{\infty} \phi(\eta) \, d\eta$  in  $x^{1/3}$  with the contribution to  $T^*(y)$  given by the in  $x^{1/3}$  with the contribution to  $T^*(y)$  given by the<br>y term in  $(\partial T^*)$  then the transformed relations are<br> $\frac{d^2 T_{\text{volume}}}{dx^2}$ 

$$
T^*(y) = T_0 + \left(\frac{\partial T^*}{\partial y}\right)_0 y + h.o.t. \tag{44}
$$

which is, of course, the same as  $(26)$  written in dimensional form, and is the standard Taylor expansion

We see that the temperature in the flowing polymer at position  $y_1$  in the unperturbed flow will be the same as the wall temperature  $T_W$  measured at a point

$$
x = \frac{hPe}{9} \left(\frac{y_1 \Gamma(\frac{2}{3})}{h}\right)^3.
$$
 (45)

A slightly more elaborate result follows if the first two terms in the expansion (43) for  $T_W - T_0$  are used.

The question now arises as to whether realistic values of x, h and Pe lead to values of  $y_1$  representative of flux balance (condition A3) dictates a solution the bulk polymer temperature. The following table shows some representative values.

$$
\begin{array}{c|cc}\nh & Pe & x & y_1 \\
(mm) & (mm) & (mm) & \text{using} \\
\hline\n2 & 5000 & 6 & 0.25\n\end{array}
$$

It is clear that the difference in temperature measured Inversion can be readily accomplished noting that by a thermocouple embedded in an insulator and one<br>by a thermocouple embedded in an insulator and one<br>touching the metal surface will only be significant when<br>the temperature gradient  $(\partial T^*/\partial y)_{0}$  is large. However touching the metal surface will only be significant when  $e^{-\mu}$   $\int_{C \cap L} P \frac{(P^{(1)} \vee p) e^{x}}{P(x)} e^{-x} dP = \frac{e^{-\mu}}{\Gamma(\frac{1}{2}) \Gamma(\frac{2}{3})} \int_{C \cap L} e^{-\mu} d\mu$ . (A6) the temperature gradient  $(\partial T^*/\partial y)_0$  is large. However, we earlier decided that values of  $1$  to  $10$  K/mm were to be expected for  $(\partial T^*/\partial y)$  and so absolute differences The inversion obtained after using result (A6), the conof order  $1 K$  in  $T_W$  are relevant. With care, such differences should be accurately detectable.

Early experiments in rubber extruders and molds shows this to be the case. It is, however. extremely difficult to prove experimentally that the wall temperature gradients predicted on the basis of the measured temperature differences are those that actually arise near metal walls because of the very inaccuracies that lead to this present method. Since the second term in equation (26) namely  $\frac{1}{2}Br\eta^2$ , becomes comparable with the first,  $\chi_0 \eta$ , within the flow channels in question, precise evaluation of so that temperature profiles within processing equipment is probably as accurately predicted by the use of complex theoretical models [I] as by temperature sensors that project into flowing streams. (A8)  $(18)$ 

- $97a^3 < 1$  (42) 1. J. R. A. Pearson, Heat transfer effects in flowing polymers. Progress in Heat and Mass Transfer, edited by W. R. Schowaher, Vol. 5. Pergamon Press, New York (1972).
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### **APPENDIX**

The problem posed by equations  $(21-25)$  can be solved by another method wholly equivalent to that presented in the section on Similarity Solutions and Series Expansion in  $\zeta$ . The equations (21-25) can be restated using a Laplace transform in the variable  $\zeta$ , provided the axial measure,  $\lambda$ ,

If

$$
T(P,\eta) = \int_0^\infty \phi(\zeta,\eta) e^{-\zeta P} dP,
$$

$$
\frac{d^2 T_{\text{polymer}}}{d\eta^2} - P\eta T_{\text{polymer}} = 0 \quad \text{for} \quad \eta > 0, \qquad (A1)
$$

$$
\frac{d^2 T_{\text{resin}}}{d\eta^2} = 0 \quad \text{for} \quad \eta < 0,\tag{A2}
$$

for  $T^*$ .<br>We see that the temperature in the flowing polymer at  $\eta = 0$ , and<br>We see that the temperature in the flowing polymer at  $\eta = 0$ , and

$$
\frac{d T_{\text{polymer}}}{d \eta} = \alpha \frac{d T_{\text{resin}}}{d \eta} - \chi_0 \quad \text{at} \quad \eta = 0,
$$

and  $0 \le \zeta < \infty$ . (A3)

The solution to equation (A1) can be expressed as a sum of Airy functions

$$
T_{\text{polymer}}(P,\eta) = aAi(P^{1/3}\eta) + bBi(P^{1/3}\eta) \quad \text{for} \quad \eta \ge 0. \tag{A4}
$$

Boundedness as  $\eta \rightarrow \infty$  requires  $b = 0$ . The solution in the lower half plane provides a solution linear in  $\eta$ . The heat

$$
T_{\text{polymor}}(P,\eta) = \frac{3^{1/3} \Gamma(\frac{1}{3}) \chi_0}{P} \frac{Ai(P^{1/3}\eta)}{\left(P^{1/3} + \frac{\alpha \Gamma(\frac{1}{3})}{3^{1/3} \Gamma(\frac{2}{3})}\right)} \quad \text{for} \quad \eta \ge 0
$$
\n
$$
\frac{y_1}{\left(P^{1/3} + \frac{\alpha \Gamma(\frac{1}{3})}{3^{1/3} \Gamma(\frac{2}{3})}\right)} \quad \text{for} \quad \eta \ge 0
$$
\n
$$
(A5)
$$

2 5000 6 0.25 
$$
Ai(0) = 3^{-2/3}/\Gamma(\frac{2}{3}), \quad Ai'(0) = 3^{-1/3}/\Gamma(\frac{1}{3}).
$$

$$
\int_{C-i\infty}^{C+i\infty} \frac{Ai}{P}(P^{1/3}v) e^{P\zeta} dP = \frac{3^{1/3}}{\Gamma(\frac{1}{3})\Gamma(\frac{2}{3})} \int_{(\frac{P^3}{Q^2})^{1/3}}^{\infty} e^{-t^3} dt. \quad (A6)
$$

volution theorem, and expanding in powers of  $x$  yields the solution

$$
\phi_{\text{polymer}}(\zeta, \eta) = \frac{3^{1/3} \chi_0}{\Gamma(\frac{1}{3}) \Gamma(\frac{2}{3})} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k}{3^{k/3}} \left( \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})} \right)^k \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{1}{4}k + \frac{1}{4})}
$$

$$
\times \int_0^{\zeta} (\zeta - t)^{(k-2)/3} \int_{\left(\frac{\eta^3}{9t}\right)^{1/3}}^{\infty} e^{-q^3} dq dt. \quad (A7)
$$

As  $\eta \rightarrow 0$ , we can recover the result of equation (41) since

$$
\int_0^{\infty} (\zeta - t)^{(k-2)/3} dt \int_0^{\infty} e^{-q^3} dq = \frac{\Gamma(\frac{4}{3})}{(\frac{1}{3}k + \frac{1}{3})} \zeta^{(k+1)/3}
$$

$$
\phi_{\text{polymer}}(\zeta, 0) = \frac{\chi_0(9\zeta)^{1/3}}{\Gamma(\frac{2}{3})} \sum_{k=0}^{\infty} \frac{(-1)^k}{3^{k/3}} \left(\frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}\right)^k \frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{1}{3}k + \frac{4}{3})} (\alpha^3 \zeta)^{k/3}.
$$

# MESURES DE TEMPERATURE A L'AIDE DE THERMOCOUPLES EN PAR01 POUR UN ECOULEMENT DE POLYMERE FONDU--I. THEORIE

Résumé-On suppose réalisé un profil de vitesse plan établi localement indépendant des variations de la température de paroi. Deux cas sont considérés pour les conditions aux limites thermiques sur la paroi: (i) paroi parfaitement conductrice c'est a dire temperature fixee et (ii) paroi faiblement conductrice, c'est a dire temperature proportionnelle au gradient de temperature. Les variations de temperature de paroi qui se produisent en aval d'un changement de la condition (i) à la condition (ii) sont évaluées en fonction de la distance aval, pour des valeurs élevées du nombre de Péclet. L'application à la mesure de la temperature moyenne d'ecoulements de polymeres est discutee.

# TEMPERATURMESSUNG DURCH WANDTHERMOELEMENTE AN POLYMERSCHMELZFLÜSSEN. I-THEORIE

Zusammenfassung-Ein voll ausgebildetes ebenes Geschwindigkeitsprofil, das örtlich unabhängig von Änderungen der Wandtemperatur ist, wird vorausgesetzt. Für die Randbedingungen an der Wand werden zwei Falle betrachtet: (i) vollkommene Leitung, d.h. feste Temperatur; und (ii) schlechte Warmeleitung, d.h. Temperatur proportional dem Temperaturgradienten. Die Änderung der Wandtemperatur, die sich ergibt bei einer Anderung der Randbedingung (i) zur Randbedingung (ii) wird als Funktion des Abstandes in Strömungsrichtung für große Werte der Peclet-Zahl berechnet. Die Anwendung auf Messungen der Mitteltemperatur von Polymerströmen wird diskutiert.

# ИЗМЕРЕНИЕ ТЕМПЕРАТУРЫ В ПОТОКЕ РАСПЛАВЛЕННОГО ПОЛИМЕРА С ПОМОЩЬЮ ВМОНТИРОВАННЫХ В СТЕНУ ТЕРМОПАР - I. ТЕОРИЯ

Аннотация - Делается допущение о полностью развитом плоском профиле скорости, локально **He3aBRCHMOM OT H3MeHeHAii TeMIIepaTypbI CTeHKH.** PaccMarpasaroTc5r nsa cnyyan rennonoro граничного условия на стенке: (1) Случай абсолютно проводящей стенки, т. е. когда темпера-**Typa CTCHKH IIOCTOIlHHa A (2) CJIy'lati IInOXO llpOBOAfllUe8 CTeHKH,T.e. KOrAa TeMllepaTypa CTeHKU**  пропорциональна температурному градиенту. Изменение температуры стенки вниз по потоку при переходе от условия (1) к условию (2) рассчитывается как функция расстояния вниз по потоку для больших чисел Пекле. Обсуждается использование этих результатов для измерения средне-массовой температуры потоков полимера.